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STATIC FORMATIONS IN THE GENERAL THEORY OF RELATIVITY AND PLANCKEONS

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Static homogeneous formations in general relativity theory are considered. It is shown that two types of formations exhaust the possible collection of such formations. The data obtained are to present the planckeon hypothesis of elementary particle structure.

The canonical form of general relativity theory, also known as canonical gravodynamics, variants of which have recently been developed by several authors [1 - 3], permits correct formulation of the general covariant definition of the intrinsic energy of an isolated object, provided the distortion of space-time for which it is responsible, is local. This condition is fulfilled with a high degree of accuracy by elementary particles. It turns out that the intrinsic energy of elementary particles in a gravitational field is finite, and that the domain of definition of an elementary particle must contain a gravitationally self-compensated domain of dimensions $L \sim 10^{-34}$ cm (or, from dimensionality considerations, 10^{-33} cm). The energy included in this domain is on the order of 10^{28} eV (10^{-5} g). The gravitational self-compensation condition implies that the dimensions L of this domain must equal its gravitational radius r_g .

On the other hand, we know the Planckian length $L^* \sim 10^{-33}$ cm and the characteristic Planckian mass $m^* \sim 10^{-56}$ g. Blokhintsev has pointed out that a mass $m^* \sim 10^{-3}$ g localized in L^* would have a gravitational radius $r_g = L^*$. Staniukovich has further noted that for such a mass $r_g = a$, where a is the radius of inner curvature. From this we see that if such an object did, in fact, exist, its characteristics would satisfy those of an object of the universe class, $r_g = L^* = a$. Staniukovich [4 and 5] has put forward the hypothesis that such objects (planckeons) do exist in our universe as relic particles left over from the instant of birth of the universe itself.

Markov [6 and 7] has suggested that such objects may exist as component parts of elementary particles, i. e. as a species of quarks of infinitely large mass (maximons). Three (or more) maximons are strongly bound to form a single system (an elementary particle) with a mass defect equal to within 10^{-20} to the characteristic rest energy of a maximon. The uncompensated rest energy is the observed energy of the elementary particle.

There exists another approach to the interpretation of planckeons with L^* and m^* . In fact, planckeons are objects of the universe class.

A planckeon, being immersed in external space-time, would have no way of declaring its existence; to an outside observer the mass of a planckeon with $L^* = r_g = a$ would be identically equal to zero.

However, the condition $L^* = r_g = a$ is violated because of fluctuations of the fields in the ambient space, and the planckeon passes into a nonideally closed state.

Part of the matter constituting a planckeon exceeds the limits of the gravitational radius and becomes explicitly observable, i. e. direct interaction between the excess matter and objects in external space becomes possible. According to the theory of Staniukovich, the excess mass of a planckeon is approximately equal to the nucleon mass in the first approximation of perturbation theory; in the third approximation it is equal to the graviton mass (the Zel'dovich-Novikov semiclosed worlds are an illustration of perturbed planckeons).

Proceeding from this model, we can represent an elementary particle as matter distributed in the localization domain of the particle which is held in by the gravitational field of the planckeon core situated deep in the center of the particle. The experimentally determined density distribution in adrons points to the possibility of a definite class of motions of the planckeon core. The virtual character of the densities does not contradict the above hypothesis.

As a first approximation we can assume that planckeons are static in the closed state and consider their possible collection.

1. Let us consider the interior problem for an ideal fluid which conforms to the energy-momentum tensor

$$T_i^k = (p + \epsilon) u_i u^k + p \delta_i^k \quad (1.1)$$

within the framework of general relativity theory,

$$R_i^k - 1/2 \delta_i^k R = \kappa T_i^k \quad (1.2)$$

We shall seek the resulting space-time metric in the form

$$- ds^2 = - c^2 dt^2 + e^{\lambda(r,t)} dr^2 + e^{\nu(r,t)} (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (1.3)$$

By a standard procedure we find from field equations (1.2) that $e^{\lambda(r,t)}$, $e^{\nu(r,t)}$, $p(r,t)$ and $\epsilon(r,t)$ satisfy the system

$$\frac{1}{4} e^{-\lambda} v'^2 - \frac{3}{4} v'' - v'' - e^{-\nu} = \kappa \left[(p + e) \frac{u^2}{c^2 \theta^2} + p \right] \quad (1.4)$$

$$e^{-\lambda} \left(\frac{1}{4} v'^2 + \frac{1}{2} v'' - \frac{1}{2} v' \lambda' \right) - \left(\frac{v''}{2} + \frac{v''}{2} + \frac{v' \lambda'}{4} + \frac{\lambda''}{4} + \frac{\lambda''}{2} \right) = \kappa p \quad (1.5)$$

$$e^{-\lambda} \left(\frac{3}{4} v'^2 + v'' - \frac{v' \lambda'}{2} \right) - \left(\frac{v''}{4} + \frac{v' \lambda'}{2} + e^{-\nu} \right) = \kappa \left(\rho - \frac{p + e}{\theta^2} \right) \quad (1.6)$$

$$v'' + \frac{v' v'}{2} - \frac{\lambda' v'}{2} = \kappa (p + e) \frac{u}{c \theta^2} e^{1/2 \lambda} \quad (1.7)$$

Let us consider homogeneous formations. This enables us to write the single equation of state

$$p = p(e) \quad (1.8)$$

The condition of comovement of the reference frame and moving matter in considering Eqs.

$$T^k_{i;k} = \frac{1}{\sqrt{-g}} \left(\frac{\partial T^k_i}{\partial x^k} \sqrt{-g} \right) - \frac{1}{2} \frac{\partial g_{\alpha\beta}}{\partial x^i} T^{\alpha\beta} = 0 \quad (1.9)$$

or, in expanded form,

$$\frac{p' + e' u^2 / c^2}{p + e} + \frac{(p' + e') u}{(p + e) c} e^{1/2 \lambda} + \frac{2 u u'}{c^2 \theta^2} + \frac{u}{c} \left(v' e^{1/2 \lambda} + \frac{u}{c} v' \right) + \frac{u'}{c \theta^2} e^{1/2 \lambda} \left(1 + \frac{u^2}{c^2} \right) + \left(\frac{u}{c} e^{1/2 \lambda} \lambda' \right) = 0 \quad (1.10)$$

$$\frac{(p' + e') u}{(p + e) c} + \frac{p' u^2 / c^2 + e'}{p + e} e^{1/2 \lambda} + \frac{\lambda'}{2} \left(1 + \frac{u^2}{c^2} \right) e^{1/2 \lambda} + \left(v' e^{1/2 \lambda} + \frac{u}{c} v' \right) + \frac{u'}{c \theta^2} \left(1 + \frac{u^2}{c^2} \right) + \frac{2 u u'}{c^2 \theta^2} e^{1/2 \lambda} = 0 \quad (1.11)$$

implies the necessity of a zero pressure gradient $p' = 0$.

In fact, the time lines in the synchronously comoving reference frame are geodesic (this is easy to verify directly). However, not only the pulverized matter moves along the geodesic lines; the same applies to all matter for which $p' = 0$, since the action of a force is always directional and must be expressed by a pressure gradient, and not by pressure itself. In other words, only matter for which the condition $p' = 0$ does not hold will "depart" from the geodesic lines. The pressure itself plays the role of a homogeneous and isotropic background imposed on the matter (*).

Recalling relation (1.8) and the condition $p' = 0$, we transform system (1.4) - (1.7) into

$$4e R_{\lambda}(r) - R_{\nu}(r) = R_{\lambda'}(r) R'_{\nu}(r) - 2R''_{\nu}(r), \quad 3/4 T'^2(t) - 3A e^{-T(t)} = \kappa e A e^{-T(t)} - 3/4 T'^2(t) - T''(t) = \kappa p \quad (1.12)$$

Interval (1.3) then becomes

$$- ds^2 = - c^2 dt^2 + e^{T(t)} [e^{R_{\lambda}(r)} dr^2 + e^{R_{\nu}(r)} d\Omega^2] \quad (1.13)$$

Complemented by equation of state (1.8), Eqs. (1.12) completely define the dynamics of the Friedmann universe.

Eq. (1.12) contains as special cases the equations for determining the form of the space part of the interval which were used by Einstein and McVittie. In fact, let us set

$$e^{R_{\nu}(r)} = e^{R_{\lambda}(r)} r^2$$

in (1.13).

*) In the most recent edition of FIELD THEORY by Landau and Lifshits (5th edition, 1967), Lifshits makes the imprecise statement that a synchronously comoving reference frame exists only if $p = 0$ (p. 377) and later (p. 389) cites Tolman's solution as the only possibility of existence of a synchronously comoving reference frame.

In this special case (1.12) becomes the familiar Eq.

$$\text{whose solution yields } \frac{\partial^2 R_\lambda(r)}{\partial r^2} = \frac{1}{r} \frac{\partial R_\lambda(r)}{\partial r} + \frac{1}{2} \left(\frac{\partial R_\lambda(r)}{\partial r} \right)^2 \tag{1.14}$$

$$e^{R_\lambda(r)} = \frac{1}{(1 + 1/4 kr^2)^2}, \quad e^{R_\nu(r)} = \frac{r^2}{(1 + 1/4 kr^2)^2} \tag{1.15}$$

With allowance for (1.15), we can rewrite interval (1.13) as

$$-ds^2 = -c^2 dt^2 + e^{T(t)} \left[\frac{dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)}{(1 + 1/4 kr^2)^2} \right] \tag{1.16}$$

This is the standard way of writing the Friedmann interval. Let us find another expression for interval (1.16). We set $\exp R_\lambda(r) = r^2$. From the expression

$$\exp R_\nu(r) = \frac{R_\nu'^2(r)}{4[A - e^{-R_\nu(r)}]}$$

which follows from Eq. (1.12), we obtain

$$\exp R_\lambda(r) = \frac{1}{1 + Ar^2} = \frac{1}{1 \pm r^2/a_0^2} \tag{1.17}$$

With allowance for (1.17) we can rewrite interval (1.13) as

$$-ds^2 = -c^2 dt^2 + e^{T(t)} \left[\frac{dr^2}{1 \pm r^2/a_0^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right] \tag{1.18}$$

which in turn can be rewritten as

$$-ds^2 = -c^2 d\tau^2 + \frac{dr^2}{1 - r^2/a^2} + r^2 d\Omega^2 = -ds^2 a_0^2/a^2$$

It is important to note here the existence of an initial radius of curvature $a_0 > 0$ ($a = a_0$ for $t = 0$). Eqs. (1.12) complemented by the equation of state $p = p(\epsilon)$ have an infinite number of solutions of which we choose the subclass of solutions with $R = \text{const}$. This is a necessary, but not sufficient, condition for constant-curvature spaces.

For static spaces we also have the condition

$$p = \text{const}, \quad \epsilon = \text{const} \tag{1.19}$$

Let us stipulate that $R = \kappa(\epsilon - 3p) = \text{const}$ in (1.12) and obtain the general solution $e^{T(t)} = A_1 \exp(\sqrt{1/3} R ct) + A_2 \exp(-\sqrt{1/3} R ct) - 6A/R$ (1.20)

Imposing additional condition (1.19) on this solution, we obtain two spaces, namely

$$-ds^2 = -c^2 dt^2 + \frac{dr^2}{1 - r^2/a_0^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \tag{1.21}$$

with the equation of state $3p + \epsilon = 0$, and

$$-ds^2 = -c^2 dt^2 + \exp(\sqrt{1/3} R ct) (dr^2 + r^2 d\theta^2 + \sin^2 \theta d\varphi^2) \tag{1.22}$$

with the equation of state $p + \epsilon = 0$. Conversion from metric (1.22) to the form

$$-ds^2 = -c^2 dt^2 \left(1 - \frac{r^2}{a_0^2} \right) + \frac{dr^2}{(1 - r^2/a_0^2)} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

and back is sufficiently simple.

The uniqueness of static worlds (1.21) and (1.22) can be demonstrated by considering the equation of hydrostatic equilibrium in general relativity theory,

$$\frac{d}{dr} \left[\frac{(p + \epsilon)(1 + \kappa r^2 p)}{(p + \epsilon) - \epsilon r dp/dr} \right] = 1 - \kappa r^2 \epsilon \tag{1.23}$$

Let $\epsilon = \epsilon_0 = \text{const}$, Eq. (1.23) then yields

$$\kappa r^2(p + \epsilon)(3p + \epsilon) = -2r(dp/dr)(1 - \kappa r^2 \epsilon)$$

This implies that when $p = \text{const}$, we have either $3p + \epsilon = 0$ or $p + \epsilon = 0$.

A difficulty arises with equations of state in which either p or ε must be negative. This difficulty has a history of its own. It was first encountered by Einstein. To avoid negative quantities, he introduced the additional term Λ in the initial equations of general relativity theory.

Negative pressure has also been used by other authors, who introduced it in arbitrary fashion in order to eliminate singularities in dynamic models of the universe. An alternative procedure is to introduce the components of the true tensor of the gravitational field, whose energy density is negative, into the energy-momentum tensor of an ideal fluid.

Introduction of such a tensor has nothing to do with the energy-momentum pseudotensor of the gravitational field obtained by the Noether theorem.

The gravitational field pseudotensor essentially describes gravitational waves and the possibility of multipolar radiation of gravitational waves. The true gravitational field tensor is a four-function of the curvature, and its trace is "indestructible" under all coordinate and reference frame transformations. In this case it is always possible to compute the gravitational field pressure in such a way that the pressure and density of matter are positive. Weyl and Eddington, who suggested varying other Lagrangians in addition to the ordinary Einstein gravitational field Lagrangian $R / 2\kappa$, were apparently right.

Without imposition of the additional conditions $p = \text{const}$ and $\varepsilon = \text{const}$, general solution (1.20) defines models of nonstatic worlds. This can be shown simply enough by verifying directly the impossibility of reducing any particular cases of an interval with the metric determined from (1.20) for any values of the integration constants A_1, A_2, A to time-independent form.

The resulting nonstatic universes expands in such a way that the average curvature in all two-dimensional directions remains constant at each point. This is accompanied by smooth transition of the equation of state from physical to "nonphysical" form. For example, matter in a universe of the form

$$- ds^2 = - c^2 dt^2 + \text{sh} \sqrt{1/3} R ct (dr^2 + r^2 d\theta^2 + \sin^2 \theta d\varphi^2)$$

has the equation of state $p - \varepsilon = 0$ (i.e. the equation of state of an ultrarelativistic gas) at the initial instant. With the passage of time the equation of state gradually becomes $p + \varepsilon = 0$.

2. We can show that universes (1.21) and (1.23) exhaust the collection of static spaces of constant curvature. To do this we consider homogeneous and isotropic formation satisfying the cosmological principle, i.e. having the metric

$$- ds^2 = - c^2 dt^2 + e^{T(t)} du^2 \tag{2.1}$$

where du^2 is the metric of a three-dimensional space hypersurface; the metric coefficients in du^2 do not depend on the time coordinate.

Among the spaces of class (2.1) we seek the subclass of spaces of constant curvature for which

$$R_{iklm} = K \begin{pmatrix} g_{il} & g_{km} \\ g_{im} & g_{kl} \end{pmatrix}$$

or, in expanded form

$$\frac{1}{2} \left(\frac{\partial^2 g_{im}}{\partial x^k \partial x^l} + \frac{\partial^2 g_{kl}}{\partial x^i \partial x^m} - \frac{\partial^2 g_{il}}{\partial x^k \partial x^m} - \frac{\partial^2 g_{km}}{\partial x^i \partial x^l} \right) + g (I_{kl}^n \Gamma_{im}^p - I_{km}^n \Gamma_{il}^p) = K \begin{pmatrix} g_{il} & g_{km} \\ g_{im} & g_{kl} \end{pmatrix} \tag{2.2}$$

The Riemann-Christoffel tensor R_{iklm} has 21 essential components. The subscripts of these components run through the following values:

1212	1223	1324	1423	2323	2424	1313
1213	1314	1334	1424	2324	2434	1314
1214	1223	1414	1434	2334	3434	1323

The components with more than two distinct subscripts yield identical zero in the right side of Eq. (2.2). There remain six essential components for which the right side is not identically equal to zero, namely

$$1212 \quad 1313 \quad 1414 \quad 2323 \quad 2424 \quad 3434 \tag{2.3}$$

The six components of (2.3) expressed in terms of the metric coefficients of the basic quadratic form (2.1) yield a system of six equations whose general solutions are spaces of constant curvature.

Let us take metric (2.1) in the form (1.18). Then

$$g_{11} = \frac{e^{T(t)}}{1 + Ar^2}, \quad g_{22} = e^{T(t)} r^2, \quad g_{33} = e^{T(t)} r^2 \sin^2 \theta \tag{2.4}$$

and the Christoffel symbols are

$$\Gamma_{22}^1 = -\frac{k'}{2h}, \quad \Gamma_{11}^1 = \frac{h'}{2h}, \quad \Gamma_{22}^0 = \frac{k'}{2}, \quad \Gamma_{11}^0 = \frac{h'}{2} \tag{2.5}$$

Substituting (2.4) and (2.5) into (2.2), we obtain an equation for the subscripts 1212, $-g_{11}(\Gamma_{22}^1 \Gamma_{11}^1) - g_{00}(\Gamma_{22}^0 \Gamma_{11}^0) = Kq_{11}q_{22}$ or $T'^2(t) - 4Ae^{-T(t)} = 4K$ (2.6)

The equation for the subscripts 1313 is of the form

$$\begin{aligned} & \frac{1}{2} \left(-\frac{\partial^2 g_{33}}{\partial x^1 \partial x^1} \right) + g_{33} (\Gamma_{13}^3 \Gamma_{13}^3) - g_{11} (\Gamma_{33}^1 \Gamma_{11}^1) = \frac{1}{2} \left(-\frac{\partial^2 e^{T(t)} r^2}{\partial r^2} \right) + \\ & + e^{T(t)} r^2 \left(\frac{k'^2}{4h^2} \right) - \frac{e^{T(t)}}{1 + Ar^2} \left(-\frac{k'h'}{4h^2} \right) + \frac{k'h'}{4} = K \frac{e^{T(t)}}{1 + Ar^2} e^{T(t)} r^2 \end{aligned}$$

After some minor transformations we again arrive at Eq. (2.6). The equation for indices 2323 yields a similar Eq. (2.6). This is to be expected, since the space part is a subspace of constant curvature, and all of the space coordinates are equivalent.

After simple transformations, Eq. (2.6) for the components with the subscripts 1414, 2424, 3434 yields Eq. $T'^2(t) + 2T''(t) = 4K$ (2.7)

For determining the function $e^{T(t)}$ we have the two equations (2.6) and (2.7), whose solution in the case $K = 0$ is $e^{T(t)} = A(ct + c_1)^2$

i. e. the special case of an expanding Euclidean space. Setting $A = 0$ in (2.6), we obtain the space-time metric for the de Sitter world. Using the substitution

$$1/y = 4 A e^{-T(t)}$$

we can reduce Eq. (2.6) to the form

$$y'^2 = 4Ky^2 + y$$

whose only solution is

$$e^{T(t)} = \begin{cases} (A_0 / K_0) \operatorname{ch}^2 \sqrt{K_0} (ct + c_1), & K = K_0 > 0 \\ (A_0 / K_0) \cos^2 \sqrt{K_0} (ct + c_1), & K = -K_0 < 0 \end{cases} \tag{2.8}$$

Solutions (2.8) resemble the expressions for the metrics of ordinary surfaces of constant curvature written out to within bending in a geodesic coordinate system. It is easy to verify that solution (2.8) also satisfies Eq. (2.7). We therefore have two homogeneous and isotropic spaces of constant curvature,

$$-ds^2 = -c^2 dt^2 + \frac{A_0}{K_0} \operatorname{ch}^2 \sqrt{K_0} (ct + c_1) \left(\frac{dr^2}{1 - r^2/a_0^2} + r^2 d\Omega^2 \right) \quad (2.9)$$

$$-ds^2 = -c^2 dt^2 + \frac{A_0}{K_0} \operatorname{cos}^2 \sqrt{K_0} (ct + c_1) \left(\frac{dr^2}{1 + r^2/a_0^2} + r^2 d\Omega^2 \right) \quad (2.10)$$

The universes defined by metrics (2.9) and (2.10) satisfy the cosmological principle, and thus conform to relations (1.12). Substituting (2.8) into (1.12), we obtain the equation of state $p + \varepsilon = 0$.

The metric of the required spaces is readily reducible to time-independent form.

Here (2.9) becomes $-ds^2 = -c^2 dt^2 \left(1 - \frac{r^2}{a_0^2} \right) + \frac{dr^2}{(1 - r^2/a_0^2)} + r^2 d\Omega^2$, (2.11)

and (2.10) becomes

$$-ds^2 = -c^2 dt^2 \left(1 + \frac{r^2}{a_0^2} \right) + \frac{dr^2}{1 + r^2/a_0^2} + r^2 d\Omega^2$$

The above notation is interesting in that the space part is written in the standard way, as for the Friedmann universes, so that several new forms, including

$$-ds^2 = -c^2 dt^2 + \frac{A_0}{K_0} \operatorname{ch}^2 (\sqrt{K_0'} ct + c_1) \left[\frac{dr^2 + r^2 d\Omega^2}{(1 + 1/4 kr^2)^2} \right]$$

can be written automatically for the de Sitter world.

Thus, there can be only two types of universes identifiable with static planckons. Planckons are, in fact, nonstatic, and the next approximation to be considered is the approximation of oscillating objects of the universe type.

This approximation already offers real hope of finding the spectrum of levels of the planckon core of elementary particles, which is related to the spectra of elementary particles.

The hypothesis of planckon structure of elementary particles has several extremely fruitful implications whose exact proof is a subject in quantum gravodynamics. One such implication is the conclusion concerning the latent energy of elementary particles, which exceeds the observed rest mass by twenty orders of magnitude.

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